

## RESEARCH PROBLEMS

With Volume 36 of Discrete Mathematics, a Research Problem Section has been established. Problems in this section are intended to be research level problems rather than standard exercises. People wishing to submit such problems should send them (in duplicate) to:

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The following should be included: (1) The name of the person(s) who originally posed the problem; (2) the name and address of a person willing to act as a correspondent; and (3) references and other pertinent information.

The Editorial Board of Discrete Mathematics invites readers to provide information about solutions, partial results and other pertinent items related to problems posed earlier, if possible indicating the source of the information, for example papers appearing in different journals, preprints, etc. This information will be passed along to readers from time to time in order to keep them apprised of the current status of various problems.

People wishing to provide information about problems that appeared earlier should write to Professor Alspach. People wishing to correspond on technical matters concerning a problem should write to the correspondent.

### **Problem 9.** Posed by Derek Holton.

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If  $X$  is a graph, let  $\text{Aut}(X)$  denote its automorphism group. Let  $X$  be a graph such that  $\text{Aut}(X)$  acts transitively on both the vertex-set and edge-set and, furthermore, if an automorphism  $\sigma_1$  maps the edge  $uv$  onto the edge  $u'v'$  such that  $\sigma_1(u) = u'$  and  $\sigma_1(v) = v'$ , then every automorphism  $\sigma$  mapping  $uv$  onto  $u'v'$  must satisfy  $\sigma(u) = u'$  and  $\sigma(v) = v'$ . Is it the case that  $\text{Aut}(X)$  must be imprimitive?

**Reference**

- [1] I.Z. Bouwer, Vertex and edge transitive, but not 1-transitive graphs, *Canad. Math. Bull.* 13 (1970) 231–237.

**Problem 10.** Posed by Douglas B. West and Michael Saks.

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Let  $P$  and  $Q$  be two partially ordered sets. The *direct product*  $P \times Q$  has for its elements all ordered pairs  $(p, q)$  with  $p \in P$  and  $q \in Q$ . It is partially ordered by  $(p, q) \leq (p', q')$  if and only if  $p \leq p'$  and  $q \leq q'$ . A *semiantichain* in  $P \times Q$  is a collection of elements in which no pair is related if they are identical in exactly one coordinate. A *unichain* in  $P \times Q$  is a chain where one coordinate remains constant, that is, the product of an element of one set with a chain in the other set. Since a unichain contains at most one element from any semiantichain, the largest cardinality of a semiantichain is less than or equal to the smallest cardinality of a unichain covering. Is it the case that equality always holds?

**References**

- [1] C. Greene and D.J. Kleitman, The structure of Sperner  $k$ -families, *J. Combin. Theory (Ser. A)* 20 (1976) 41–68.  
[2] D.B. West and C.A. Tovey, Semiantichains and unichain coverings in direct products of partial orders, *SIAM J. Algebraic Discrete Methods* 2 (1981).